Observation of Evolutionary Velocity Field in Matching Pennies Game

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In a matching pennies game of laboratory experimental economics, with the metric for an instantaneous velocity measurement, we report the first observation of the evolutionary velocity field in the population strategy state space.

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Background: In evolutionary game theory (EGT), an evolutionary dynamic equation is a story about a velocity field of evolution. A velocity field describes the direction and the magnitude of the populations strategy state change ($\triangle x$) in a unit of time ($\triangle t$) at each given state x in the populations strategy state space $\mathbb{X}$ [1–4].

Laboratory experimental economics removes EGT from its abstract setting and links the theory to observed behavior [5]. The observed populations behavior in the experiments are systematic, replicable, and roughly consistent with the dynamical systems approach [6–12]. However, the evolutionary velocity field has never been obtained empirically. In this letter, using a laboratory matching pennies game (MPG) as an example, we explore the evolutionary velocity field.

Space, State and Time: In EGT analogy [4], without loss of generality, we use a two-population MPG as an example. In the first population X, the strategy set is $\{X_1,X_2\}$ for each agent; similarly, in the second population Y, $\{Y_1,Y_2\}$. The payoff matrix is in Table I [14]. If there are 4 agents in each population, an observable instantaneous populations strategy state should be $x{:=}(p,q) \in \mathbb{X}$, herein $\mathbb{X}$ is the populations strategy state space and $\mathbb{X}{=}\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\} \bigotimes \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$, and p (q) is the density of X_1 (Y_1) in X (Y). Figure 1 is an illustration, in which the square 5×5 lattice (gray dots) is the space $\mathbb{X}$ and $x_A{=}(\frac{1}{4}, \frac{3}{4})$ is a state.

In experimental economics, the time t is the label along the repeated rounds in a session. At each round t, there is one observation of x_t in $\mathbb{X}$. The *smallest* tick $\triangle t$ is 1 and is the time interval between two closest rounds.

TABLE I: Payoff matrix of the MPG

	Y_1	Y_2
X_1	5, 0	0, 5
X_2	0, 5	5, 0

Metric for Velocity: Now, we introduce our metric for the evolutionary velocity. Without loss of generality,

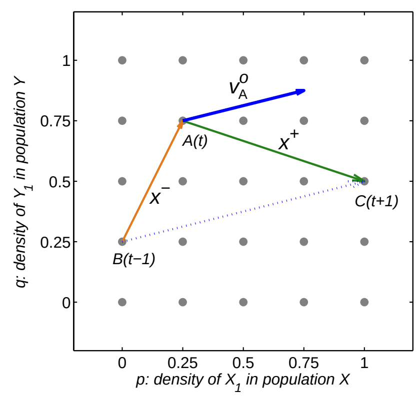

FIG. 1: Schematic diagram for an observation of an instantaneous velocity v^o_{pq}. The 5×5 Lattice (in gray dots) is the population strategy state space $\mathbb{X}$ of the two populations MPG. x_A is one of the states in $\mathbb{X}$ and $x_A{=}(0.25, 0.75)$. Suppose that, during a microscope process $\mathbb{P}$ including both a forward change (x^+) and a backward chang (x^-), the observed state at t is x_A, and at $(t-1)$ is x_B and at $(t+1)$ is x_C, by Eq. (1), $v^o_A{=}(x_C - x_B)/2$ (arrow in blue) is an instantaneous velocity at x_A.

Figure 1 shows a microscope process $\mathbb{P}$ from which one observation of *an instantaneous velocity*, v^o_{pq}, can be defined. From t to t+1, the forward change at a given state (p, q) is described as $x^+{=}x_{t+1} - x_t$; meanwhile, from t-1 to t, the backward change is $x^-{=}x_t - x_{t-1}$. Hence, at a given state (p, q), we define, v^o_{pq}, as

$$v^o_{pq} := \left(x^+ + x^-\right)/(2\triangle t), \tag{1}$$

in which $\triangle t$=1, practically, in experimental economics. In Eq.(1), v^o_{pq} satisfies two requirements of the measurement for instantaneous velocity (1) instantaneous: time interval is practically the smallest; (2) time odd: it is time reversal asymmetric [15]. Then, the mean velocity vector at the (p, q) can be computed as

$$\overline{v}_{pq} = \frac{1}{\Omega_{pq}} \sum_o v^o_{pq}, \tag{2}$$

where Ω_{pq} is the total observation of the state (p, q) and the summation is carried over all of the rounds when-

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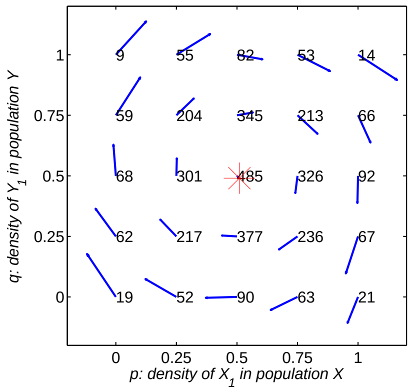

FIG. 2: Obtained evolutionary velocity field in laboratory MPG. The red star ($*$) indicates the mean observation of density. The numeric label indicates the observed occupation Ω_{pq} of each state. The blue arrow from each state indicates the direction and the magnitude of mean velocity $\overline{v}_{pq}$. The plotting ratio is $1:1$.

ever the observed state is (p,q). A simple and practical example see Table II.

Experiment: As the traditional experiment setting for social evolution [6–12], we employed the MPG among two populations with the payoff matrix in Table I. Each population includes 4 subjects. There are 12 independent sessions and each session includes 300 rounds repeated with random matching protocol. For more details, see [16].

Results: Figure 2 shows the main results from our data of MPG, in which (1) the red star indicates the mean observation of density is x^{*}=$(0.510, 0.490)$; meanwhile for each state (2) the numeric label indicates the total observation Ω_{pq} and (3) the blue arrow indicates the mean velocity vector $\overline{v}_{pq}$.

For state (p,q), denote the *normal vector* as r_{pq}:=$x_{pq}-x^{*}$, statistically, we have $|\overline{v}_{pq}|$ positively depends on $|r_{pq}|$ (p=0.000); meanwhile, the angle from $\overline{v}_{pq}$ to r_{pq} is a right angle ($\pm 7\%$) [17].

Summary: In our data of MPG, statistically, the larger the magnitude of the normal vector is, the faster the evolution should be; meanwhile, the direction of the evolution is perpendicular to the normal vector. In full population strategy state space, the evolutionary velocity field pattern is cyclic and clockwise.

As a conclusion, in data from a laboratory matching pennies game, this letter demonstrates (1) the empirical existence of the evolutionary velocity field pattern and (2) the practical metric for an instantaneous velocity.

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[14] In each cell, the numeric in up-left is the payoff for the agent from X and down-right for the agent from Y.

[15] Not lost generality, as shown in Fig.1, if $\mathbb{P}^T$ is the time reversal process of $\mathbb{P}$, the path of $\mathbb{P}^T$ remains the same as $\mathbb{P}$ but the direction of propagation is reversed [13], e.g., from C to A to B, and so, with Eq. (1), the v^o_A in $\mathbb{P}^T$ should be $(x_B - x_C)/2$ and is the opposite to v^o_A in $\mathbb{P}$, so v^o_{pq} is time reversal asymmetry. In ref. [10], in the measurement for the change of state in a given state, the backward change x^- term is not included and the measured variable $\triangle\mathbf{x}/\triangle t$ in ref. [10] is not time odd.

[16] Experiment procedure and records. The experiment includes 12 independent sessions. Each session involves 8 human subjects split randomly into two populations no matter what gender of a subject is. Each populations includes 4 subjects. The role of subjects in one population is called as player X and in the other, player Y. Once a subject's role is generated it is persisted on during a session, meanwhile, we only tell the subjects the payoff matrix. Each session consists of 300 rounds of the game repeatedly with a random matching for each round. The experiments were conducted with a computerizing controlled environment. Each subject sat in an isolated seat

TABLE II: Mean velocity $\overline{v}_{pq}$ calculated from the instantaneous velocity vectors of a given state x_{pq}[§]

ID	Session	Round (t)	x_{t-1}	$x_{(0,1)t}$	x_{t+1}	x^-	x^+	$v^o_{(0,1)}$
1	1^α	63^β	0.50, 1.00^γ	0, 1^γ	0.25, 1.00^γ	-0.50, 0.00	0.25, 0.00	-0.125, 0.000
2	1	274	0.25, 0.25	0, 1	0.75, 1.00	-0.25, 0.75	0.75, 0.00	0.250, 0.375
3	8	204	0.25, 0.50	0, 1	0.75, 0.75	-0.25, 0.50	0.75, -0.25	0.250, 0.125
4	9	133	0.25, 0.75	0, 1	0.50, 0.75	-0.25, 0.25	0.50, -0.25	0.125, 0.000
5	9	233	0.25, 0.25	0, 1	0.75, 1.00	-0.25, 0.75	0.75, 0.00	0.250, 0.375
6	10	67	0.25, 0.25	0, 1	0.50, 1.00	-0.25, 0.75	0.50, 0.00	0.125, 0.375
7	10	182	0.25, 0.50	0, 1	0.25, 0.50	-0.25, 0.50	0.25, -0.50	0.000, 0.000
8	11	292	0.00, 0.50	0, 1	0.50, 0.50	0.00, 0.50	0.50, -0.50	0.250, 0.000
9	12	283	0.50, 0.25	0, 1	0.75, 0.50	-0.50, 0.75	0.75, -0.50	0.125, 0.125
Summary: at the state $(0,1)$, the total observation of $\Omega_{(0,1)}$ is 9 and the mean velocity $\overline{v}_{(0,1)}$ vector is (0.139, 0.153)								

§ The given state x_{pq} is $(0,1)$ in this example. In data, whenever the $x_t = (1, 0.5)$ state is obtained, one more record is appended to this table. As total 9 records obtained, the $\Omega_{(0,1)}$ is 9. In the last line, the vecor $\overline{v}_{(0,1)}$=(0.139, 0.153) is the velocity vector at the top-left state in Figure 2.

• Column x_{t-1} and x_{t+1} is the state of the last round and the state of the next round of x_t, respectively, and both of which can be archived from raw data.

• Column x^+ and x^- are the forward change and the backward change, respectively.

• Column $v^o_{(0,1)}$ is one observation of the instantaneous velocity at the state $(0,1)$ obtained at the round (in 3-rd column) and the session (in 2-nd column).

• Data comes from all of the 3600 records in the 12 session and the first round and the last round is excluded as a given state in each session. For the top and the end round are unavailable for x^+ or x^-.

$^\alpha$ The serial number of a session, number 1 denotes the 1st session in the 12 sessions in our experiment.

$^\beta$ The serial round number in the session, e.g., the number 63 denotes the 63rd round.

$^\gamma$ Numeric pair (p,q) includes the density (p) of X_1 in X and then the density (q) of Y_1 in Y, respectively, and can be calculated from the records of individual decision making in each round.

with a computer. No communication among was allowed during the experiment. The software for the experiments was developed as a Web base system by the authors. There is not any computer system problem in this experiment. In each session, the player X has two options, X_1 and X_2, while the player Y has Y_1 and Y_2. The player X selected to use 'X_1' or 'X_2' button and player Y selected to use 'Y_1' or 'Y_2' button simultaneously. After each round one received the feedback including his or her own strategy used in the previous round, his or her opponent's strategy in the previous round, and his or her own payoff from the previous round. The payoff is calculated according the payoff matric in Table I. The subjects were asked to document the information on his/her experimental records. At the end of the experiments, one could obtain the accumulated score which would be changed into RMB currency, as the subject's payments after a session. The exchange rate is each 1 experimental points for 0.07 Yuan RMB, in addition, every participant was paid 5 Yuan RMB as the showup fee.

We conducted the laboratory experimental sessions in the Experimental Social Science Laboratory of Zhejiang University in Hangzhou China in October 26 and 27, 2010. Each session lasts about 2 hours including the introduction stage in both the written and oral forms to inform the participating subjects the game protocol as well as a test drive for the subjects to be familiarized with the game and the experiments. There were in total 96 undergraduate students of Zhejiang University majoring in different areas recruited into these experiments with each subject participating in only one session. The average payoff for each participant was 57.50 Yuan RMB. We collect 28800 records of individual decision making in the 12 sessions with 300 rounds and 8 subjects each.

[17] Statistical analysis.

For the relationship of $|v_{pq}|$ and $|r_{pq}|$, the statistical model of the simple linear regression is $|v_{pq}| = \beta\,|r_{pq}| + \alpha$, where the samples are the 25 states cover all (p,q) frequency weighted by Ω_{pq}, and the OLS regression results are β=0.276±0.001 and α=0.007±0.000.

For the angle θ_{pq} from $\overline{v}_{pq}$ to r_{pq} and θ_{pq}:=arccos($\mathbf{r}_{pq} \bullet \overline{v}_{pq})/(|\mathbf{r}_{pq}||\overline{v}_{pq}|)$, , we have (1) using the 25 states cover all (p,q) as the samples, in mean±s.e. format θ_{pq}=1.614±0.062; (2) using the 25 states cover all (p,q) as the samples but excluding the state (0.5,0.5), from *t-test* with *Ho*: mean=$\pi/2$, the results are mean value is 1.557, *Std. Err.*=0.0282, *Std. Dev.*=0.138 and the 95% Conf. Interval is in $[1.499, 1.615]$ and (3) from *ci* test with weighted Ω_{pq} but excluding the state (0.5,0.5), the results are mean value is 1.584, *Std. Err.*=0.0226 and the 95% Conf. Interval is in $[1.538, 1.631]$. We use software package Stata 10 to archive above results.