Laboratory RPS Game Experimental Evaluations of the Evolutionary Dynamics Theories (Summary)

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1. Theory

The evolution equation determines how frequencies of strategies in the population change (Traulsen & Hauert (2009)), where the change indicates velocities (Bowles (2004),Weibull (1997),Sigmund (2010)). In this paper, based on the laboratory Rock-Paper-Scissors (RPS) game experiments and from the velocity vector fields perspective, we empirically study the five most popular evolutionary dynamics theories (Sandholm (2009)), i.e., replicator(Taylor & Jonker (1978)), logit (Fudenberg & Levine (1998)), Brown-von Neumann-Nash (Brown & Von Neumann (1950)) (BNN), best responde (Gilboa & Matsui (1991)) and Smith (Smith (1984)), respectively, which describe different theoretical patterns of the velocity vector fields in strategy space.

2. Experiment

We conduct a standard Rock-Paper-Scissors (RPS) game in an experimental economics laboratory. The experiments include 12 independent sessions and each session involves 6 human subjects. Each session consists of 300 rounds of the game repeatedly with a random matching for each round. Since we have 6 human subjects in the RPS game experiments, we have 28 possible states. With the data collected from the experiments, we measure the velocity vector at all of the 28 measurable states $x_{(i,j,k)}$ in the strategy space of the game.

3. Result

We have the following observations and findings.

(1)The velocity vector fields obtained in the experiments fall into a cyclic pattern with the velocity amplitudes linearly dependent upon the Euclidian distances between the state positions and the center of the triangle of the strategy space. The linear frequency weights regression of speed $|\upsilon_{(i,j,k)}|$ on $|x_{(i,j,k)} - \bar{x}_0|$ results are $0.126 \pm .016$ with the *const*. $= 0.002 \pm .005$. Figure 1(a) is the empirical velocity vector field pattern from our experimental data. On distribution on the strategy states, the mean observation of the three strategy of Rock-Paper-Scissors are $\bar{x}_{obs} = (x_R; x_P; x_S) = (0.322, 0.356, 0.322)$ respectively, and this result is fair close to all of the five theoretical prediction, $\bar{x}_0 = (x_R; x_P; x_S) = (1/3, 1/3, 1/3)$.

(2) For the first time in the literature of the experimental economics, this paper reports an empirical study on the evolutionary dynamics theories using the velocity vector fields, which substantially advances the existing literature (Van Huyck (2008), Bouchez & Friedman (2008) and Cheung & Friedman (1998)). Further, it is verified experimentally for the first time that the vector field magnitudes in the strategy space follow the properties consistent with those of the existing theories. Figure 1 graphically documents the findings from the laboratory study on the evolutionary dynamics theories based on the RPS game experiments focusing on the velocity vector fields, where the 28 velocity

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Table 1: Quantitative comparison between the velocity vector field patterns in the triangle strategy space obtained from the laboratory RPS game experiments and the five classic evolutionary dynamics theories, respectively. $*$ labeled numbers indicate the best match of the corresponding model with the experimental data.

Dynamics	Angle Similarity (AS)	Magnitude Similarity (MS)	Field Similarity (FS)
$Replicator$	0.102051^*	0.016397^*	0.787061^*
$Logit[0.08]$	0.240512	0.014387	0.631025
$SampleBR[2]$	0.226466	0.012258	0.549449
BNN	0.176287	0.010299	0.631876
$Smith$	0.107017	0.013302	0.770103

vectors obtained from the RPS game experiments as well as their corresponding theoretic counterparts in each of the five comparing classic evolutionary dynamics theories are overlaid on the triangle strategy space within which each positional point is encoded by a color value to represent the magnitude of the velocity vector at this position with the blue value indicating the minimum velocity magnitude and the red value indicating the maximum velocity magnitude. In Figure 1, (a) shows the velocity vector distribution in the triangle strategy space obtained from the experimental study, and (b)–(f) show the corresponding theoretic velocity vector distributions in the triangle strategy space from the classic evolutionary dynamics theories of the replicator, the logit, the best response, the BNN, and the Smith, respectively. From this figure, it is clear that none of the five classic theories is completely consistent with what the RPS experimental study shows. Specifically, the experiments support and verify well the replicator model in the sense that both the experimental and theoretic velocity vector field distributions in the triangle strategy space are a cyclic pattern. Further, in terms of the magnitude value distributions of the velocity vector field in the triangle strategy space, the experiments imply that the replicator and the Smith models are closer to the experimental data than the other theoretic models. This finding is significant as it is the first in the literature to report such empirical study for the classic evolutionary dynamics theories, though the experimental data still appear a bit counter-intuitive in particular for the velocity vector magnitude values at the three corners of the triangle that are lower after the weighting of the frequency than what they should be intuitively, which requires a further investigation.

(3) In order to further quantitatively compare the five classic theories with the experimental results, we define three metrics to measure the difference between the experimental result and a specific model. The first metric is called Angle Similarity (AS), which is defined as the summation of the 28 angles of the pairs of the corresponding velocity vectors between the experimental data and the theoretic data from the comparing model; a smaller AS indicates a higher similarity between the two patterns. The second metric is called the Magnitude Similarity (MS); here we take the magnitudes of the 28 velocity vectors as a 28 dimensional vector and then define the cosine of the angle between the two 28 dimensional vectors of the experimental data and the theoretic data from a comparing model as the MS value; a larger MS indicates a higher similarity between the two patterns. The third metric is called the Field Similarity (FS) and is defined as the summation of the 28 cosine values of the individual angles between the corresponding velocity vectors of the experimental data and the theoretic data, respectively. In all the above three metrics, we also consider the frequency information in the experiments as the weights of the related vectors into the definitions. Table 1 documents the comparison results between the experimental results and each of the five classic models using all the three metrics. From the table, it is clear that AS and FS are consistent in measuring the differences. Also it is clear that the replicator model beats all the other four models in the sense that it is the closest to the experimental data all consistent from the three metrics.

4. Conclusion

This is the first time in the literature of experimental economics to reveal the global dynamic patterns in the strategy space using laboratory RPS game study. This is also the first time in the literature to apply the velocity vector fields to study the comparison between the experimental data and the classic evolutionary dynamics theories, which for the first time to show that none of the classic theories completely confirms the experimental data. This is once again the first time in the literature to report the empirical evaluations on evolutionary dynamics using the human subject data in the decision making experiments.

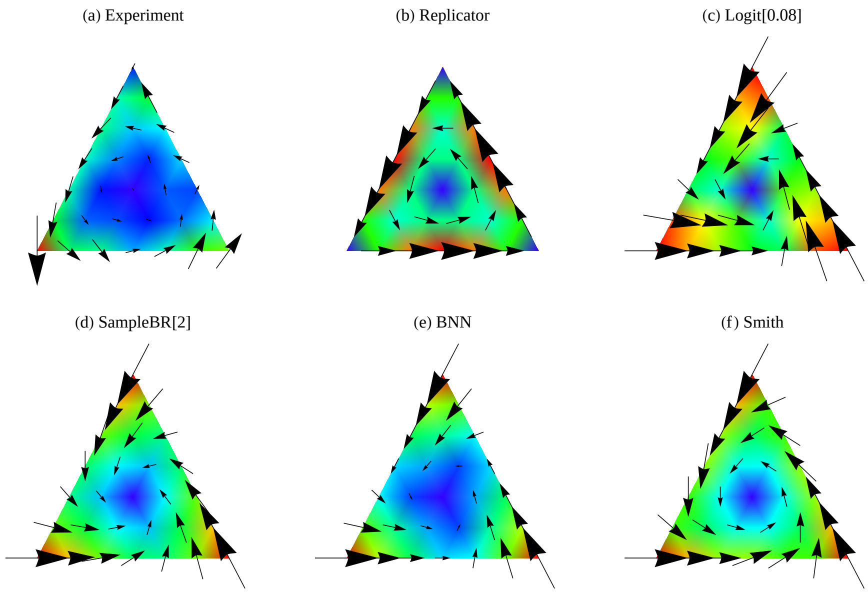

Figure 1: Graphical comparison between the velocity vector field patterns in the triangle strategy space (with population size N=6 and 28 possible observable states) obtained from (a) the laboratory RPS game experiments and (b-f) the five classic evolutionary dynamics theories, replicator(Taylor & Jonker (1978)), Logit (Fudenberg & Levine (1998)) with $\lambda = 0.08$, Sample best responde (Gilboa & Matsui (1991)) with $k = 2$, Brown-von Neumann-Nash, BNN (Brown & Von Neumann (1950)) and Smith (Smith (1984)), respectively. (b)-(f) are calculated with *Dynamo* 3, for one-population, three strategy games Sandholm & Dokumaci (2007), specified on the 28 strategies sites which can be measured from our experimental data.

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